Exercise 18

For the following exercises, determine whether there is a minimum or maximum value to each quadratic function. Find the value and the axis of symmetry.

$$h(t) = -4t^2 + 6t - 1$$

Solution

Begin by factoring the coefficient of t^2.

$$h(t) = -4\left(t^2 - \frac{3}{2}t + \frac{1}{4}\right)$$

In order to write this quadratic function in vertex form, it's necessary to complete the square, which makes use of the following algebraic identity.

$$(t + B)^2 = t^2 + 2tB + B^2$$

Notice that $2B = -\frac{3}{2}$, which means $B = -\frac{3}{4}$ and $B^2 = \frac{9}{16}$. Add and subtract $\frac{9}{16}$ on the right side within the parentheses and use the identity so that x appears in only one place.

$$\begin{aligned} h(t) &= -4\left[\left(t^2 - \frac{3}{2}t + \frac{9}{16}\right) + \frac{1}{4} - \frac{9}{16}\right] \\ &= -4\left[\left(t + \left(-\frac{3}{4}\right)\right)^2 - \frac{5}{16}\right] \\ &= -4\left(t - \frac{3}{4}\right)^2 + \frac{5}{4} \end{aligned}$$

Therefore, the vertex of the parabola is $\left(\frac{3}{4}, \frac{5}{4}\right)$. The axis of symmetry is $t = \frac{3}{4}$, and the maximum (because the coefficient of t^2 is negative) is $y = \frac{5}{4}$.